DYNAMIC RESPONSE TO MOVING CONCENTRATED LOADS OF SIMPLY SUPPORTED PRE-STRESSED BERNOULLI-EULER BEAM RESTING ON BI-PARAMETRIC SUBGRADES

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ABSTRACT: The dynamic analysis of uniform prestressed Bernoulli-Euler beam resting on bi-parametric subgrades and traversed by concentrated loads having simple support ends conditions is investigated in this paper. The solution technique is based on the Finite Fourier Sine transform with the series representation of the Dirac-delta function, a modification of Struble's asymptotic method and Laplace transformation in conjunction with convolution theory. Analytical solution and numerical analysis showed that higher values of axial force N, shear modulus G and Foundation modulus K, reduced the response amplitudes of the beam when is under the action of moving concentrated loads. However, higher values of shear modulus G are required for a more noticeable effect than the values of foundation modulus K. Also, the critical speed for the system traversed by moving force is found to be smaller than that under the influence of moving mass, hence resonance is reaches earlier in the moving mass problem than that of the moving force problem.

Keywords: Uniform Elastic beam, bi-parametric sub-grades, pre-stress, Concentrated Loads, Resonance, Moving force, Moving mass, Critical Speed

1. Introduction

The study of the dynamic response to moving loads of elastic solid bodies (beam, plates or shells) has been the concerned of several researchers in applied mathematics, physics and Engineering. By virtue of the relevance of the study in the analysis and design of railway, bridges, elevated roadways, decking slabs, the dynamics response of structural members under the passage of moving loads has been extensively investigated and a number of experimental and numerical studies have been reported in literature in recent years. Fryba [1] is an excellent book on analytical solution of moving loads over structures. Cifuentes [2] has studied the subject using auxiliary functions with finite element approximation. Wu [3] studied vibrations of a frame structure due to a moving trolley and the hoisted object. Yavari et al [4] have investigated analytical solution of dynamic response of an overhead crane system. Gbadeyan and Oni [5] consider the dynamic behaviours of beams and rectangular plates under moving loads.

Several other researchers have made tremendous feat into the study of dynamic of structures under moving loads in the recent years. These include Oni [6], Oni and Omoloje [U.S.A] [7], Oni and Awodola [8], Yuksel and Aksoy [9], Pesterer et al [10], Vostrunkhor and Metrikine [11], Nguyen [13] and Gbadeyan et al [17].

However, the above studies considered only the Winkler approximation model which has been criticized variously by Authors [14, 15, 16] because it predicts discontinuities in the deflection of the surface of the foundation at the ends of a finite beam, which is in contradiction to observations made in practice.

To this end, Coskun [17] considered the dynamic response to a harmonic load of a finite beam on tensionless two parameter foundation; Guter [18] studied the circular elastic plate resting on a tensionless Pasternak foundation under symmetric and asymmetric loading while Ma et al studied the static analysis of an infinite beam resting on a tensionless Pasternak foundation.

Thus, this paper investigated the dynamic response to moving concentrated load of pre-stressed uniform simply supported Bernoulli-Euler beam resting on bi-parametric sub-grades, in particular, Pasternak sub-grades.

2 THE INITIAL BOUNDARY-VALUE PROBLEM.

The governing partial differential equation for a uniform pre-stressed simply supported Bernoulli-Euler beam of length L on bi-parametric subgrades, in particular Pasternak subgrades and traversed by a concentrated load P(x,t) of mass M moving with velocity c is given by .

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 V(x,t)}{\partial x^2}\right)+\mu\frac{\partial^2 V(x,t)}{\partial t^2}-N\frac{\partial^2 V(x,t)}{\partial x^2}+P_G(x,t)=P(x,t) \qquad (2.1)$$

where, E is the young modulus, I is the moment of inertia, EI is the flexural rigidity of the beam, V(x, t) is the transverse deflection, μ is the constant mass per unit length of the beam, N is the constant axial force, x is the spatial coordinate taking along the axis of the beam, t is the time variable and $P_G(x,t)$ is the foundation reaction given by

$$P_G(x,t)=kV(x,t)-G\frac{\partial^2 V(x,t)}{\partial x^2} \qquad (2.2)$$

In this system, when the effect of the mass of the moving load on the transverse displacement of the Uniform Bernoulli-Euler beam is considered, the load P(x, t) takes the form

$$P(x,t)=P_f(x,t)\left[1-\frac{1}{g}\frac{d^2V(x,t)}{dt^2}\right] \qquad (2.3)$$

where the continuous moving force P_f (x,t) acting on the beam model is given as

$$P_f(x,t)=Mg\delta(x-ct) \qquad (2.4)$$

where *M* and *c* are the mass and the speed of the moving load respectively, g is the acceleration due to gravity and $\frac{d^2}{dt^2}$ is a convective acceleration operation defined as

$$\frac{d^2}{dt^2} = \frac{\partial^2}{\partial t^2} + \frac{2c\partial^2}{\partial x \partial t} + \frac{c^2\partial^2}{\partial x^2} \tag{2.5}$$

and $\delta(x-ct)$ is the Dirac delta function defined as

$$\delta(x-ct) = \begin{cases} 0, x \neq ct \\ \infty, x = ct \end{cases} \tag{2.8}$$

with the properties

$$\delta(-x) = \delta(x) \tag{2.9}$$

$$\int_a^b \delta(x-ct) f(x)dx = \begin{cases} 0, ct < a < b \\ f(ct), a < ct < b \\ 0, 0 < b < ct \end{cases} \tag{2.10}$$

In mechanics, the Dirac delta function may be thought of as a unit concentrated force acing at a point

$X = 0$

The Bernoulli-Euler beam under consideration is assumed to be uniform, which implies, the beams properties such as young's modulus E, the moment of inertia I and the mass per unit length μ of the beam do not vary throughout the span L of the beam. Using equations (2.2), (2.3), (2.4), and (2.5) in equation (2.1) and after rearrangement one obtains.

$$\begin{gathered} H_1 \frac{\partial^4 V(x,t)}{\partial x^4} + \frac{\partial^2 V(x,t)}{\partial t^2} - \lambda_0 \frac{\partial^2 V(x,t)}{\partial x^2} + H_2 V(x,t) \\ -\delta(x-ct)\left[H_3 \frac{\partial^2 V(x,t)}{\partial t^2} + H_4 \frac{\partial^2 V(x,t)}{\partial x \partial t} + H_5 \frac{\partial^2 V(x,t)}{\partial x^2}\right] = P\delta(x-ct) \end{gathered} \tag{2.11}$$

where

$$H_1 = \frac{EI}{\mu}, \quad H_2 = \frac{K}{\mu}, \quad H_3 = \frac{M}{\mu}, \quad H_4 = 2cH_3, \; H_5 = c^2 H_3, \; \lambda_0 = \frac{N+G}{\mu} \tag{2.12a}$$

and $$P = \frac{Mg}{\mu} \tag{2.12b}$$

The Simply Supported boundary conditions are

$$V(0,t) = 0 = V(L,t), \; \frac{\partial^2 V(0,t)}{\partial x^2} = 0 = \frac{\partial^2 V(L,t)}{\partial x^2} \tag{2.12}$$

and the associated initial conditions are

$$V(x,0) = 0 = \frac{\partial V(x,0)}{\partial t} \tag{2.13}$$

3. SOLUTION PROCEDURE.

Equation (2.11) is a fourth order partial differential equation with variable and singular coefficients. In this section, a general approach is developed in order to solve the initial-boundary value problem. The approach involves expressing the Dirac-delta function as a Fourier cosine series and then reducing the modified form of the fourth order partial differential equation above using the Finite Fourier sine transform (3.1). The resulting couple transformed differential equation is then simplified using the modified Struble's asymptotic technique. The Finite Fourier sine transform is defined by

$$Z(m,t)=\int_0^L V(x,t)Sin\frac{m\pi x}{L}dx \quad (3.1)$$

with the inverse

$$V(x,t)=\frac{2}{L}\sum_{m=1}^{\infty} Z(m,t)Sin\frac{m\pi x}{L} \quad (3.2)$$

In order to solve equation (2.11) subject to (2.12). Thus, applying (3.1) to (2.11), we obtain.

$$\mathrm{Z_{tt}(m,t)}+(\omega_{\mathrm{m}}^2+\mathrm{H_2})\mathrm{Z(m,t)}-\lambda_0 T_A(t)+T_B(t)$$
$$+T_C(t)+T_D(t)=PSin\frac{m\pi x}{L} \quad (3.3)$$

where

$$T_A(t)=\int_0^L \frac{\partial^2 V(x,t)}{\partial x^2}Sin\frac{m\pi x}{L}dx \quad (3.4)$$

$$T_B(t)=H_3\int_0^L \delta(x-ct)\frac{\partial^2 V(x,t)}{\partial t^2}Sin\frac{m\pi x}{L}dx \quad (3.5a)$$

$$T_C(t)=H_4\int_0^L \delta(x-ct)\frac{\partial^2 V(x,t)}{\partial x\partial t}Sin\frac{m\pi x}{L}dx \quad (3.5b)$$

$$T_D(t)=H_5\int_0^L \frac{\partial^2 V(x,t)}{\partial x^2}Sin\frac{m\pi x}{L}dx \quad (3.5c)$$

and

$$\omega_m^2=\frac{m^4\pi^4}{L}H_1\ , \quad P=\frac{Mg}{\mu} \quad (3.6)$$

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In view of equation (3.2), evaluation of integrals (3.4) gives

$$T_A(t) = -\frac{m^2\pi^2}{L^2} Z(m,t) \tag{3.7}$$

In order to evaluate integrals (3.5a), use is made of the Dirac-delta function as an even function to express it as a Fourier Cosine Series namely,

$$\delta(x-ct) = \frac{1}{L} + \sum_{n=1}^{\infty} Cos \frac{n\pi ct}{L} Cos \frac{n\pi x}{L} \tag{3.8}$$

Substituting (3.8) into (3.5a), $T_B(t)$ can be rewritten as

$$\begin{aligned} T_B(t) &= \frac{2}{L^2} \sum_{k=1}^{\infty} Z_{tt}(k,t) \int_0^L Sin \frac{k\pi x}{L} Sin \frac{m\pi x}{L} dx \\ &+ \frac{2}{L^2} \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} Z_{tt}(k,t) Cos \frac{n\pi ct}{L} (N_1 - N_2) \end{aligned} \tag{3.9}$$

where

$$N_1 = \int_0^L Sin \frac{(n+k)\pi x}{L} Sin \frac{m\pi x}{L} dx \tag{3.10}$$

$$N_2 = \int_0^L Sin \frac{k\pi x}{L} Sin \frac{m\pi x}{L} dx \tag{3.11}$$

Further simplification and rearrangement yield

$$T_B(t) = \frac{M}{L\mu} [Z_{tt}(m,t) + 2\sum_{k=1}^{\infty} Z(k,t) Sin \frac{m\pi ct}{L} Sin \frac{k\pi ct}{L}] \tag{3.12}$$

Using similar argument, it is straight forward to show that

$$\begin{aligned} T_C(t) = -\frac{M}{L\mu} \{ [\frac{8ckm}{k^2 - m^2} Z_t(k,t) + 16 \sum_{n=1}^{\infty} \frac{cmk(n^2 + k^2 - m^2)}{((n+k)^2 - m^2)((n-k)^2 - m^2)}] \\ x\ Cos \frac{n\pi ct}{L} Z_t(k,t) \} \end{aligned} \tag{3.13}$$

and

$$T_D(t) = -\frac{M}{L\mu} [\frac{m^2\pi c^2}{L^2} Z(m,t) + \frac{2k^2\pi^2c^2}{L^2} Sin \frac{m\pi ct}{L} Sin \frac{k\pi ct}{L}] Z(k,t) \tag{3.14}$$

Substituting equations (3.7),(3.12),(3.13) and (3.14) into (3.3), one obtains.

$$\begin{aligned} &Z_{tt}(m,t)+\omega_{mf}^2 Z(m,t)+\alpha_0\{Z_{tt}(m,t)-\frac{m^2\pi^2c^2}{L^2}Z(m,t) \\ &+\sum_{k=1}^{\infty}[N_a(t)Z_{tt}(k,t)-N_b(t)Z_t(k,t)-N_c(t)Z(k,t)]\}=PSin\frac{m\pi x}{L} \end{aligned} \tag{3.15}$$

where

$$\alpha_0=\frac{M}{L\mu}\,,\quad \omega_{mf}^2=\frac{EI}{\mu}(\frac{m\pi}{L})^2+\frac{k}{\mu}+\lambda_0(\frac{m\pi}{L^2})^2 \tag{3.16}$$

$$N_a(t)=2Sin\frac{m\pi ct}{L}Sin\frac{k\pi ct}{L},N_b(t)=2[N_{a1}(m,k)+2N_{a2}(m,n,k)Cos\frac{m\pi ct}{L}] \tag{3.17}$$

$$N_c(t)=\frac{2c^2k^2\pi^2}{L^2}Sin\frac{m\pi ct}{L}Sin\frac{k\pi ct}{L},N_{a1}(m,k)=\frac{4mck}{k^2-m^2} \tag{3.18}$$

and

$$N_{a2}(m,n,k)=4\sum_{n=1}^{\infty}\frac{ckm(n^2+k^2-m^2)}{((n+k)^2-m^2)(((n-k)^2+m^2))} \tag{3.19}$$

Equation (3.15) represents the transformed equation of the uniform Bernoulli-Euler beam model simply supported at both ends. In the next section, we discuss two special cases of the equation.

4. ANALYSIS OF THE TRANSFORMED EQUATION

(i) Case1

Setting $\alpha_0=0$ in equation (3.15), we have

$$Z_{tt}(m,t)+\omega_{mf}^2 Z(m,t)=PSin\frac{m\pi x}{L} \tag{4.1}$$

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This represents the classical case of a moving force problem associated with our system. It is an approximate model, which assumes the inertia effect of the moving mass as negligible.

Solving equation (4.1) in conjuction with the initial conditions (2.13) and inverting gives

$$Z(x,t) = \frac{1}{L}\sum_{m=1}^{\infty}\frac{P}{\omega_{mf}}\left[\frac{\omega_{mf}\,Sin\frac{m\pi c}{L}t - \frac{m\pi c}{L}Sin\,\omega_{mf}t}{\omega_{mf}^2 - \left(\frac{m\pi c}{L}\right)^2}\right]Sin\frac{m\pi x}{L} \qquad (4.2)$$

Equation (4.2) represents the transverse displacement response to a moving concentrated force, moving at constant velocity of a uniform simply supported Bernoulli-Euler beam resting on bi-parametric sub-grades, in particular, Pasternak subgrades.

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If the moving load has mass commensurable with that of the structure, the inertia effect of the moving load is not negligible. Thus, in this case $\alpha_0 \mp 0$, and we are required to solve the entire equation (3.15). This, we term the moving mass problem. Evidently, a closed form solution of equation (3.15) is not possible.

Unlike case1, it is obvious that an exact analytical solution to this equation is not possible. Thus, we resort to an approximate analytical method which is a modification of the asymptotic method due to Struble. First, equation (3.15) is rearrange to take the form

$$\frac{d^2Z(m,t)}{dt^2} - \left(\frac{\alpha_0 R_1(t)}{R_4(t)}\right)\frac{dZ(m,t)}{dt} + \left(\frac{Q_{mf}^2 - \alpha_0 R_2(t)}{R_4(t)}\right)Z(m,t)$$

$$+\left(\frac{\alpha_0}{R_4(t)}\right)\sum_{\substack{k=1,\\ k\neq m}}^{\infty}\{N_a(t)Z_{tt}(k,t) - N_b(t)Z_t(k,t) - N_c(t)Z(k,t)\} = \left(\frac{\alpha_0 Lg}{R_4(t)}\right)\sin\frac{m\pi ct}{L} \qquad (4.3)$$

where

$$R_0(t) = 1 + 2\sin^2 \frac{m\pi ct}{L} \tag{4.4}$$

$$R_1(t) = \frac{16m^2c}{L}\sum_{n=1}^{\infty}\frac{1}{(n^2-4m^2)} Cos\frac{n\pi ct}{L} \tag{4.5}$$

$$R_2(t) = \frac{m^2c^2k^2\pi^2}{L^2}(1+\sin^2\frac{m\pi ct}{L}) \tag{4.6}$$

$$R_4(t) = 1 + \alpha_0 R_0(t) \tag{4.7}$$

Next, we consider the homogeneous part of (4.3) and obtain a modified frequency corresponding to the frequency of the free system due to the presence of the moving mass.An equivalent free system operator defined by the modified frequency then replaces equation (4.3).

In order to do this, we consider a parameter α_1 <1 for any arbitrary mass ratio α_0 defined by

$$\alpha_1 = \frac{\alpha_0}{1+\alpha_0} \tag{4.8}$$

It follows that

$$\alpha_0 = \alpha_1 + 0(\alpha_1^2) \tag{4.9}$$

and

$$\frac{1}{R_4} = \frac{1}{1+\alpha_0 R_0(t)} = 1 - \alpha_1 R_0(t) \tag{4.10}$$

Next, we substitute equation (4.10) into (4.3) to obtains

$$\begin{aligned}&\frac{d^2Z(m,t)}{dt^2} - \alpha_1 R_1(t)\frac{dZ(m,t)}{dt} + \omega_{mf}^2(1-\alpha_1 R_0(t))Z(m,t) - \alpha_1 R_2(t)Z(m,t)\\ &+\alpha_1\sum_{\substack{k=1,\\k\neq m}}^{\infty}\{N_a(t)Z_{tt}(k,t) - N_b(t)Z_t(k,t) - N_c(t)Z(k,t)\} = \alpha_1 Lg\sin\frac{m\pi ct}{L}\end{aligned} \tag{4.11}$$

When we set $\alpha_1 = 0$, we obtain a case corresponding to the case when the inertial effect of the mass of the system is neglected, the solution of (4.3) can be written as

$$Z(m,t) = \phi_m Cos(\omega_{mf}^2 t - \psi_m) \tag{4.13}$$

Where ϕ_m and ψ_m are constants.

Since $\alpha_1 < 1$, an asymptotic solution of the homogeneous part of (4.3) can be written as

$$Z(m,t) = \phi(m,t)Cos(\omega_{mf}^2 t - \psi(m,t)) + Z_1(m,t) + 0(\alpha_1^2) \tag{4.14}$$

Where $\phi(m,t)$ and $\psi(m,t)$ are slowly time varying functions. The modified frequency is obtained by substituting equation (4.14) into the homogeneous part of equation (4.3). The resulting variational equations describing the behaviour of $\phi(m,t)$ and $\psi(m,t)$ during the motion of the mass determine the modified frequency.

Thus, substituting (4.14) into the homogeneous part of (4.3) and neglecting terms which do not contribute to variational equations, we have.

$$\begin{aligned} &-2\omega_{mf}\dot{\phi}(m,t)Sin(\omega_{mf}t - \psi(m,t)) \\ &+\{2\omega_{mf}\dot{\psi}(m,t) - 2\alpha_1(\frac{m^2c^2\pi^2}{L^2} + \omega_{mf}^2)\}\phi(m,t)Cos(\omega_{mf}t - \psi(m,t)) \end{aligned} \tag{4.15}$$

Retaining terms to $0(\alpha_1)$ only.

The variational equations of our problem are obtained by setting coefficients of $Sin(\omega_{mf}t - \psi(m,t))$ and $Cos(\omega_{mf}t - \psi(m,t)$ to zero respectively. Thus, we have

$$-2\omega_{mf}\dot{\phi}(m,t) = 0 \tag{4.16}$$

$$\{2\omega_{mf}\dot{\psi}(m,t) - 2\alpha_1(\frac{m^2c^2\pi^2}{L^2} + \omega_{mf}^2)\}\phi(m,t) \tag{4.17}$$

Solving (4.16) and (4.17), one obtains

$$\phi(m,t) = B_k \tag{4.18}$$

Where B_k is a constant and

$$\psi(m,t) = \frac{\alpha_1}{\omega_{mf}L^2}(m^2c^2\pi^2 + \omega_{mf}^2 L^2)t + \psi_m \tag{4.19}$$

Therefore, when the effect of the mass of the particle is considered, the first approximation to the homogeneous system is

$$Z(m,t) = B_k Cos[\varphi_m t - \omega_m] \tag{4.20}$$

Where

$$\varphi_m = \frac{\omega_{mf}^2 L^2 - (m^2 c^2 \pi^2 + \omega_{mf}^2 L^2)\alpha_1}{\omega_{mf} L^2} \tag{4.21}$$

Is called the modified natural frequency representing the frequency of the free system due to the presence of the moving mass.

Thus, the homogeneous part of (4.3) can be written as

$$\frac{d^2 Z(m,t)}{dt^2} + \varphi_m^2 Z(m,t) = 0 \tag{4.22}$$

and equation (4.3) then takes the form

$$\frac{d^2 Z(m,t)}{dt^2} + \varphi_m^2 Z(m,t) = \alpha_1 Lg Sin \frac{m\pi ct}{L} \tag{4.23}$$

Retaining terms to 0(α_1) only.

Solving equation (4.23) in conjuction with the initial conditions and inverting we obtain

$$Z(x,t) = \frac{2}{L} \sum_{m=1}^{\infty} \frac{\alpha_1 Lg}{2\varphi_m} \left[\frac{\varphi_m Sin \frac{m\pi c}{L} t - \frac{m\pi c}{L} Sin \varphi_m t}{\varphi_m^2 - (\frac{m\pi c}{L})^2} \right] Sin \frac{m\pi x}{L} \tag{4.24}$$

This represents the transverse displacement response to a concentrated mass moving with constant velocity of simply supported prestressed uniform Bernoulli-Euler beam resting on bi-parmetric sub-grades, in particular,pasternak subgrades.

5. ANALYSIS OF RESULTS

In a dynamical problem such as this, one is interested in the resonance condition. These are the conditions under which the Bernoulli-Euler beam responses grow without bound.

Evidently, from equation (4.2), the Bernoulli-Euler beam response under a moving force will grow witnout bound whenever.

$$\omega_{mf} = \frac{k\pi c}{L} \tag{5.1}$$

While from equation (4.24), the same Bernoulli-Euler beam traversed by a moving mass encounter a resonance effect at

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$$\varphi_m = \frac{k\pi c}{L} \tag{5.2}$$

From equation (4.21) we have

$$\omega_{mf} = \frac{\omega_{mf}^2 L^2 - (m^2 c^2 \pi^2 + \omega_{mf}^2 L^2)\alpha_1}{k\pi c L} \tag{5.3}$$

It can be deduced from equation (5.3) that, for the same natural frequency, the critical speed for the system of Bernoulli-Euler beam traversed by a moving mass is smaller than that of the same system traversed by a moving force. Thus, for the same natural frequency of the Bernoulli-Euler beam, the resonance is reach earlier by considering the moving mass system than by moving force system.

6. NUMERICAL CALCULATIONS AND DISCUSSIONS OF RESULTS

In this section, numerical results for the uniform simply supported Bernoulli-Euler beam are presented in plotted curves. An elastic beam of length 12.192m is considered. Other values used are modulus of elasticity E = 2.10924 x 10^{10}N/m^2, the moment of inertia I = 2.87698 x 10^{-3}m and mass per unit length of the beam μ = 3401.563Kg/m. The value of the foundation constant *(k)* is varied between *ON/m^3 and 400000N/m^3*, the value of axial force *N* is varied between *ON* and 2.0 x 10^8N, the values of the shear modulus (G) varied between *ON/m$^{3\ and}$ 900 000N/m^3*. The results are as shown in the various graphs below.

Figure1, displays the transverse displacement response to a moving force of simply supported pre-stressed uniform Bernoulli-Euler beam for various values of axial force and for fixed value of shear modulus G and foundation moduli K. The graphs show that the response amplitudes increases as the value of the axial force decreases for fixed values of foundation stiffness K and shear modulus *G*. Figure 2 also shows the deflection profile due to moving force of a simply supported uniform Bernoulli-Euler beam for fixed value of shear modulli G, axial force N and various values of foundation modulli K. The graph shows that the response amplitudes of the beam decreases as the values of the foundation modulli K are increased.Figure3 shows the deflection profile of a simply supported Bernoulli-Euler beam for various values of shear modulus G and for fixed values of foundation modulus K and axial force N. The graph shows that increased in the values of the shear modulus reduce the response amplitudes of the beam.

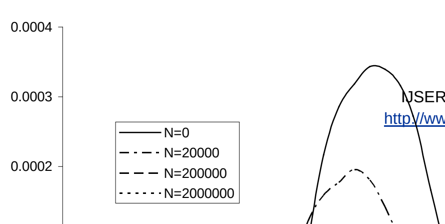

Fig.1: Deflection profile of a Simply supported uniform Bernoulli-Euler Beam under moving force for fixed values of Shear modulus (G=90000), Foundation Modulus (k=40000) and various values of Axial Force (N)

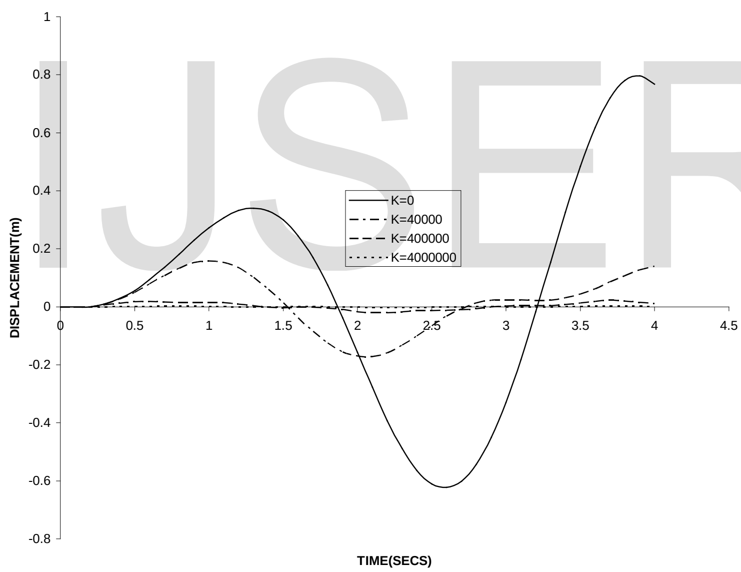

Fig.2: Deflection profile of a Simply supported uniform Bernoulli-Euler Beam under moving force for fixed values of Shear modulus (G=90000), Axial force (N=20000) and various values of Foundation modulli (K)

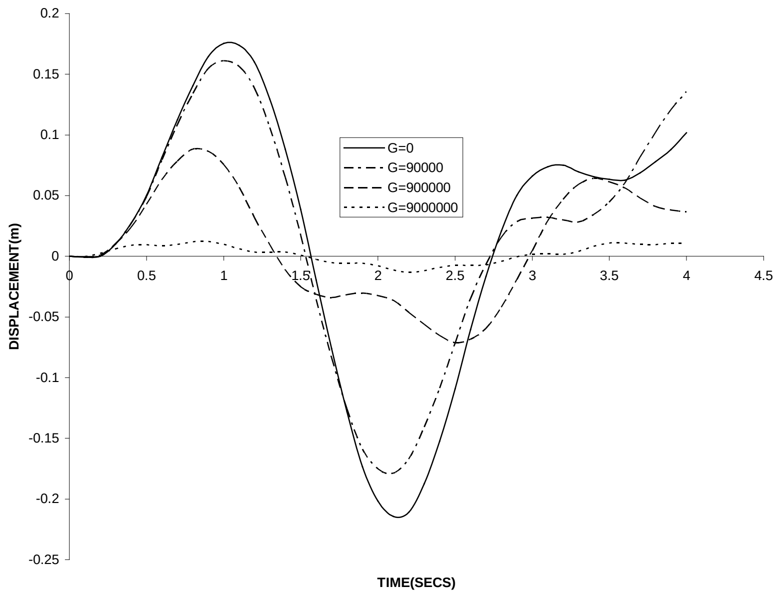

Fig 3: Deflection profile of the simply supported uniform Bernoulli-Euler beam under a moving force for various values of shear modulus G, and fixed values of axial force and foundation modulli K

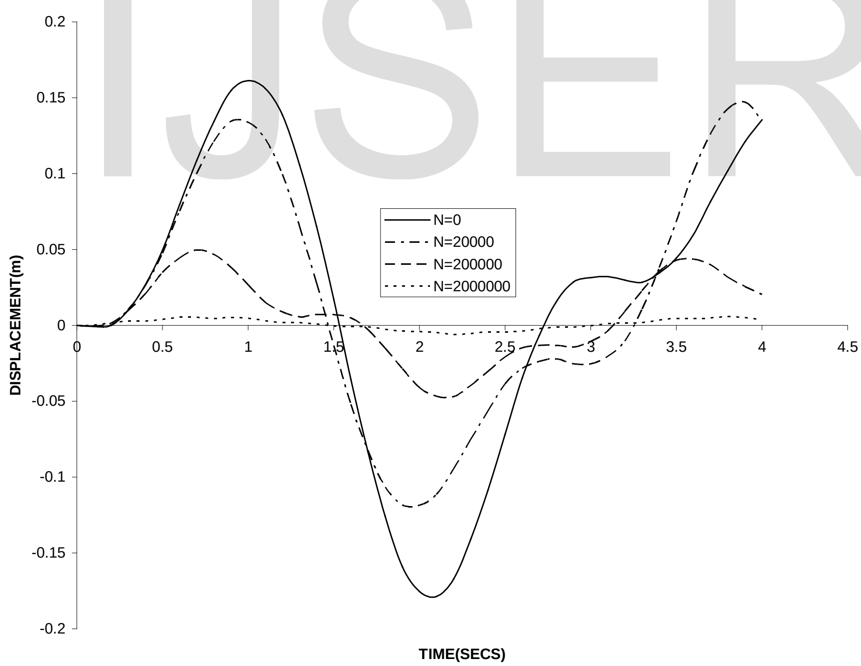

Fig 4: Deflection profile of a Simply supported uniform Bernoulli-Euler Beam under moving mass for fixed values of Shear modulus (G=90000), Foundation Modulus (k=40000) and various values of Axial Force (N)

Furthermore, Figure 5 displays the transverse displacement response of a uniform simply supported Bernoulli-Euler beam under the action of moving masses for various values of foundation modulus K and fixed values of axial force N=20000 and shear modulus G=90000.Evidently, as K increases the deflection of the uniform beam decreases. The deflection profiles of the beam for various values of the shear modulus G and for fixed values of the axial force N=20000 and foundation modulus K=40000 are shown in figure 6.It is shown that as shear modulus increases the deflection of the beam decreases.

Finally, Figure 7 depicts the comparism of the transverse displacement of the moving force and moving mass for fixed values of the axial force N=20000, shear modulus G=90000 and foundation modulus K=40000.Obviously, the graph shows that, the response amplitudes of the moving mass is higher than that of a moving force, sowing that the moving force solution is not always an upper bound for the accurate solution to a moving mass problem. This shows that relying on moving force solution could seriously be misleading and tragic.

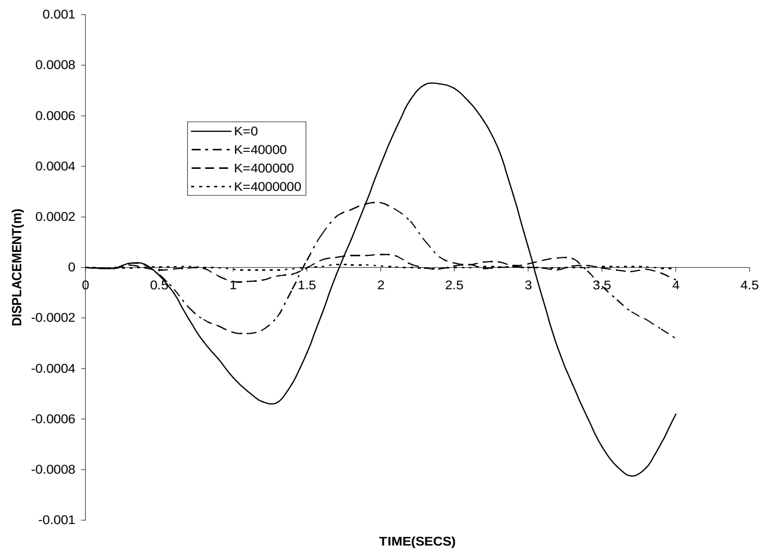

Fig 5: Deflection profile of a Simply supported uniform Bernoulli-Euler Beam under moving mass for fixed values of Shear modulus (G=90000), Axial force (N=20000) and various values of Foundation modulli (K)

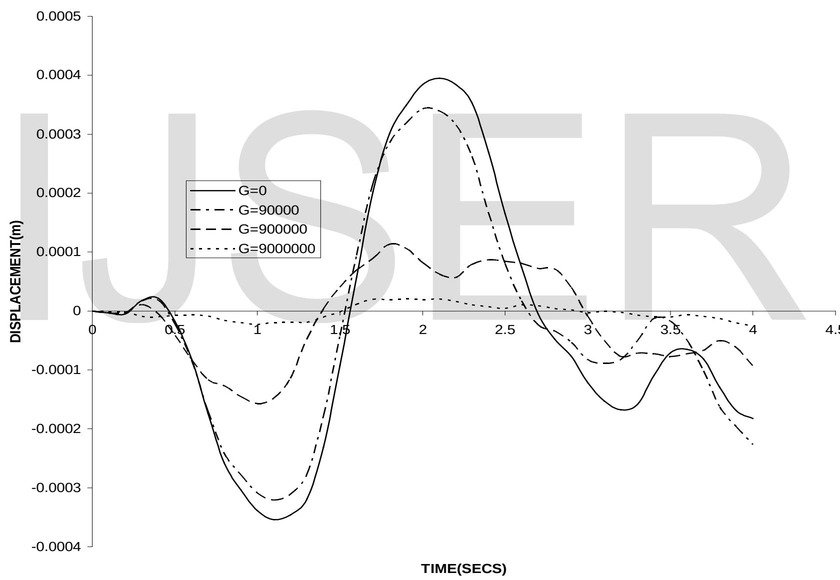

Fig 6: Deflection profile of the simply supported uniform Bernoulli-Euler beam under a moving mass for various values of shear modulus G, and fixed values of axial force and foundation modulli K

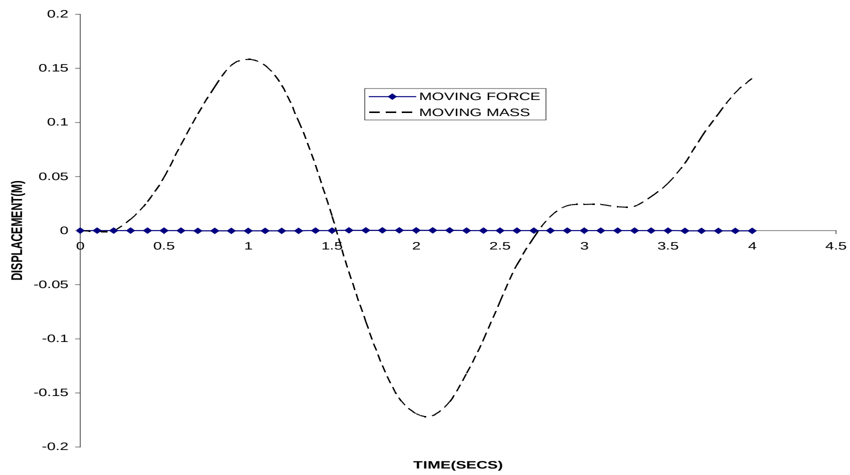

Fig 7:Comparison of the displacement response of moving force and moving mass cases for simply supported Bernoulli-Euler beam for fixed axial force N(20000),foundation modulus K(40000) and shear modulusG(90000).

7. Conclusion

In this paper, the problem of the dynamic response to moving concentrated load of a prestressed Uniform Simply Supported Bernoulli-Euler beam resting on bi-parametric subgrades, in particular, Pasternak subgrades has been solved. The approximate analytical solution technique is based on the finite Fourier sine transform, Laplace transformation and convolution theory and finally modification of the Struble's asymptotic method. Analytical solutions and Numerical analysis show that, the critical speed for the same system consisting of a pre-stressed uniform simply supported Bernoulli-Euler beam resting on bi-parametric subgrades, in particular, Pasternak subgrades and traversed by a moving mass is smaller than that traversed by a moving force and this shows that, moving force solution is not an upper bound for the accurate solution of the moving mass problem. Furthermore, an increase in the foundation modulus K with fixed values of shear modulus G and axial force N reduces the amplitudes of vibration of the beam. Also, the amplitudes of vibration decreases with an increases in the values of the shear modulus with fixed values of foundation modulus and axial force. Also, increase in the values of the axial force with fixed values of shear modulus and foundation modulus. Finally, it was observed that, higher values of shear modulus are required for a more noticeable effect than that of the foundation modulus.

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